TURKISH AVIATION ACADEMY

İTÜ

# Fleet Assignment

Dr. Peter Belobaba

Istanbul Technical University

Air Transportation Management

M.Sc. Program

Network, Fleet and Schedule

Strategic Planning

Module 20 : 1 April 2016

# *Lecture Outline*

- **Fleet Assignment Problem**
  - Objectives and principal economic trade-offs
- **Single Flight Leg vs. Network Fleet Assignment**
  - Operational constraints and modeling assumptions
  - Example: Network fleet assignment problem
- **Network Fleet Assignment Optimization**
  - Objective function and constraints
  - Solution times
- **Coldstart: Fleet Assignment at Delta Air Lines**

# *SCHEDULE DEVELOPMENT*

- **Involves several interrelated decisions, which to date have not been fully integrated:**

  Frequency Planning: Number of departures to be offered on each route, non-stop versus multi-stop

  Timetable Development: Flight departure and arrival times, including connections at airline hubs

  Fleet Assignment: Aircraft type for each flight, based on demand and operating cost estimates

  Aircraft Rotation Planning: Links consecutive flights to ensure balanced aircraft flows on the network.

# *Fleet Assignment Problem*

- **Given a schedule of flight legs (origin, destination, departure and arrival times), as well as:**
  - Number of Aircraft by Equipment Type
  - Turn Times by Fleet Type at each Station
  - Other Restrictions: Maintenance, Gate, Noise, Runway, etc.

- **Operating Costs and Spill Costs determine the Total Potential Contribution of each Flight, by Fleet Type**

- **What is the optimal (contribution/profit maximizing) assignment of aircraft to each flight leg?**

# *Trade-offs in Fleet Assignment*

- **Operating costs increase with size of airplane for any given flight (typically)**
  - Larger aircraft have higher ownership and maintenance costs
  - Increased fuel burn with greater capacity and weight
  - More (and perhaps higher paid) crew members required
- **Spill costs decrease with size of airplane**
  - SPILL is rejected demand due to inadequate capacity
  - Larger aircraft accommodate more demand and generate more revenue, meaning less spill and lower spill costs
- **Economic trade-off in choosing optimal fleet type**
  - Too large an aircraft leads to higher costs, empty seats
  - Too small an aircraft leads to higher load factors but more rejected demand and lost revenue potential

# *Fleet Assignment Example – Single Leg*

A  B

Demand = 100
Fare = $100

| Fleet Type | Capacity | Spill Cost | Op. Cost | Assignment Cost |
|---|---|---|---|---|
| i | 80 | $2,000 | $5,000 | $7,000 |
| ii | 100 | $0 | $6,000 | $6,000 |
| iii | 120 | $0 | $7,000 | $7,000 |
| iv | 150 | $0 | $8,000 | $8,000 |

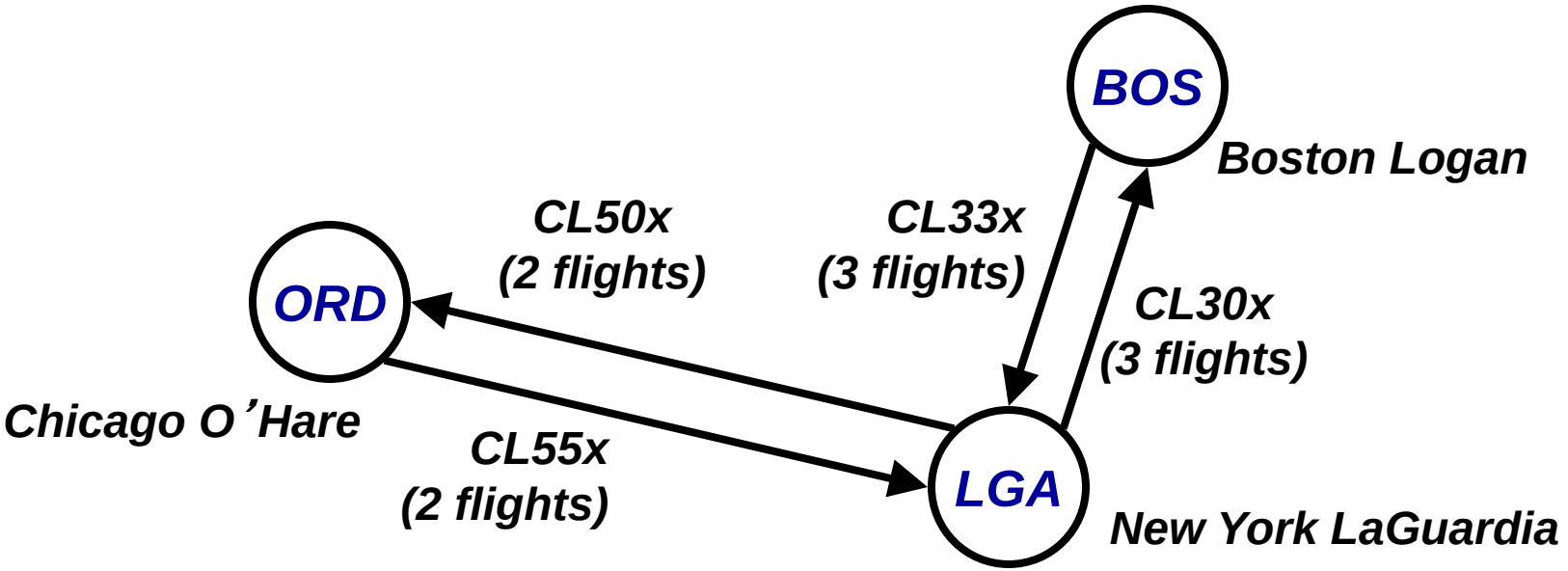
***SOURCE: Prof. C. Barnhart***

# *Single Leg vs. Network Fleet Assignment*

- **Single leg spill and fleet assignment is unrealistic:**
  - Larger aircraft must be available at origin airport at required departure time
  - Larger aircraft must return (or continue onward) from destination airport
  - Smaller aircraft must be assigned to an alternative profitable flight leg
  - Crew rotations, maintenance and other considerations
- **Also, single leg model assumes 100% local traffic**
  - Changing aircraft size on one flight will affect connecting network passenger flows on other flights
  - No recapture assumed – changing aircraft size on one flight will affect passenger loads on other flights on the same route

# Example: Network Fleet Assignment

## Flight Network




SOURCE: Prof. C. Barnhart

# Flight Schedule, Fares, & Demand

| Flight # | From | To | Dept Time (EST) | Arr Time (EST) | Fare [$] | Demand [passengers] |
|---|---|---|---|---|---|---|
| CL301 | LGA | BOS | 1000 | 1100 | 150 | 250 |
| CL302 | LGA | BOS | 1100 | 1200 | 150 | 250 |
| CL303 | LGA | BOS | 1800 | 1900 | 150 | 100 |
| CL331 | BOS | LGA | 0700 | 0800 | 150 | 150 |
| CL332 | BOS | LGA | 1030 | 1130 | 150 | 300 |
| CL333 | BOS | LGA | 1800 | 1900 | 150 | 150 |
| CL501 | LGA | ORD | 1100 | 1400 | 400 | 150 |
| CL502 | LGA | ORD | 1500 | 1800 | 400 | 200 |
| CL551 | ORD | LGA | 0700 | 1000 | 400 | 200 |
| CL552 | ORD | LGA | 0830 | 1130 | 400 | 150 |

***9***

***SOURCE: Prof. C. Barnhart***

# *Example: Network Fleet Assignment*

## Fleet Information

| Fleet type | Number of aircraft owned | Capacity [seats] | Per flight operating cost [$000] | |
|---|---|---|---|---|
| | | | LGA - BOS | LGA – ORD |
| DC-9 | 1 | 120 | 10 | 15 |
| B737 | 2 | 150 | 12 | 17 |
| A300 | 2 | 250 | 15 | 20 |

*SOURCE: Prof. C. Barnhart*

# Example: Network Fleet Assignment

## Evaluating assignment profits…

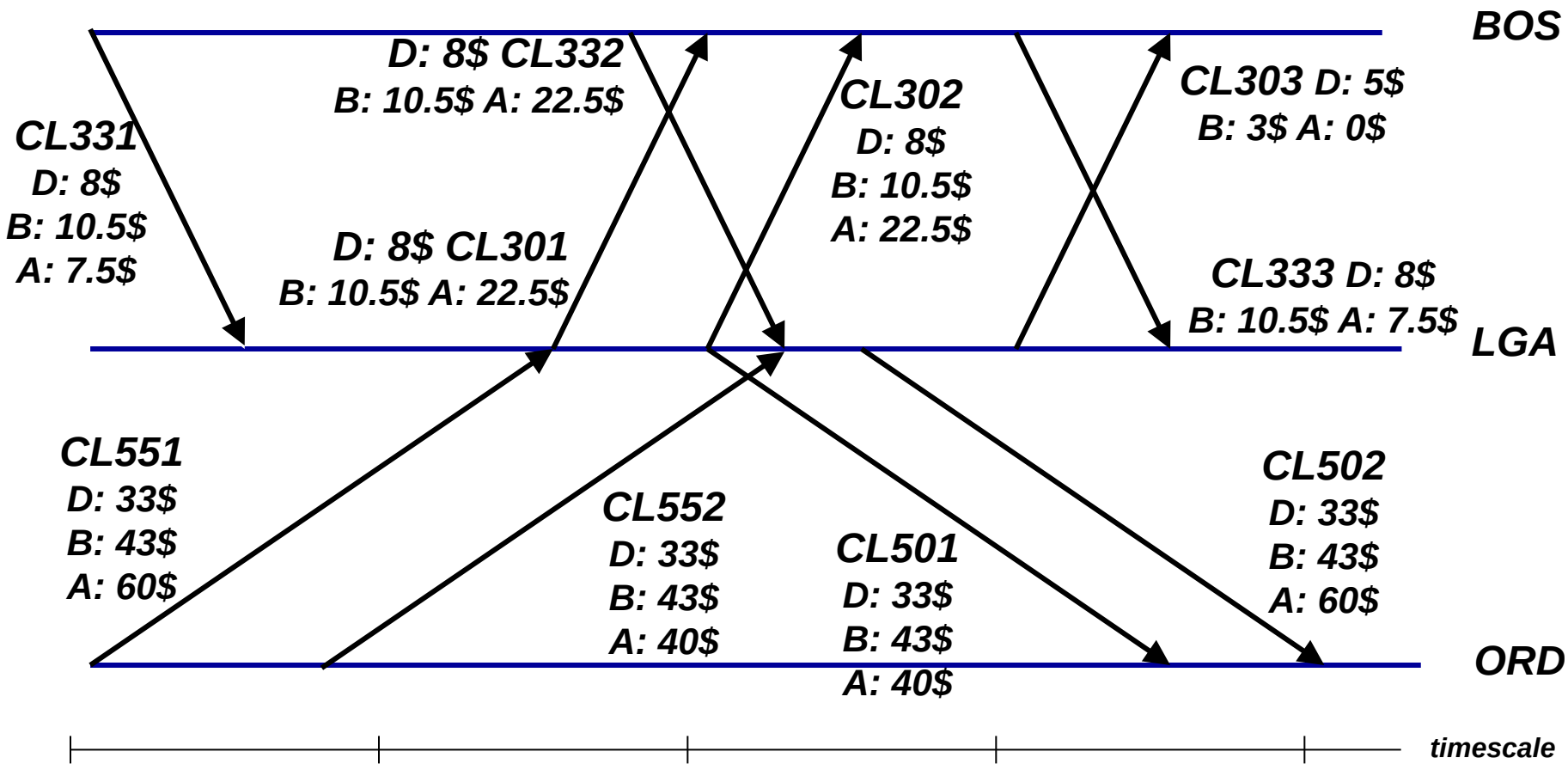
*Profitability [$000 per day]*

| Flight # | DC-9 | B737 | A300 |
|---|---|---|---|
| CL301 | 8 | 10.5 | 22.5 |
| CL302 | 8 | 10.5 | 22.5 |
| CL303 | 5 | 3 | 0 |
| CL331 | 8 | 10.5 | 7.5 |
| CL332 | 8 | 10.5 | 22.5 |
| CL333 | 8 | 10.5 | 7.5 |
| CL501 | 33 | 43 | 40 |
| CL502 | 33 | 43 | 60 |
| CL551 | 33 | 43 | 60 |
| CL552 | 33 | 43 | 40 |

*SOURCE: Prof. C. Barnhart*

# Example: Network Fleet Assignment

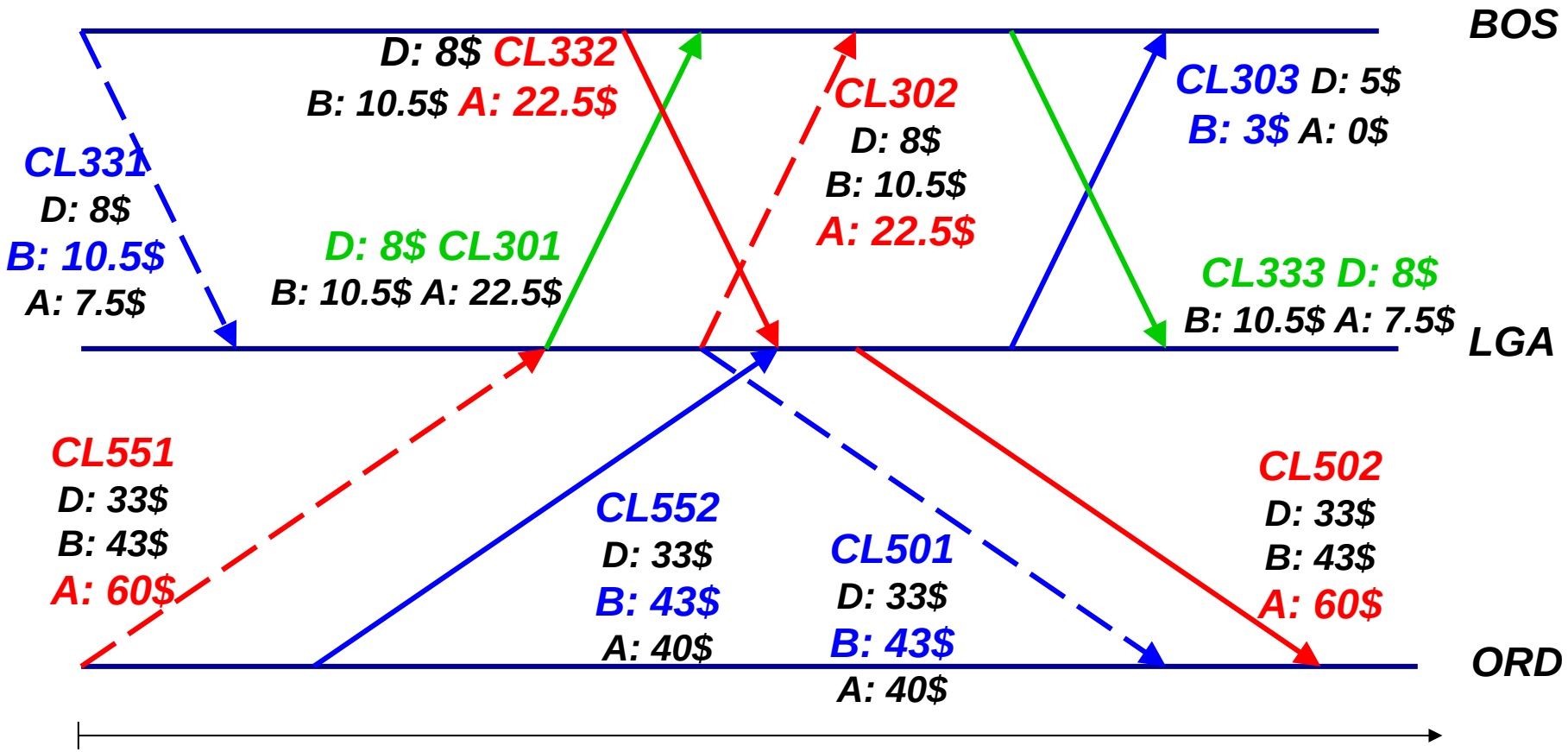
## Assign 1 DC9, 2 B737 and 2 A300 to Time-Line Network:




*SOURCE: Prof. C. Barnhart*

# Fleet Assignment Solution

## Assign 1 DC9, 2 B737 and 2 A300 to Time-Line Network:




**Revenue = $428,500  Cost = $148,000  Profit = $280,500**

*SOURCE: Prof. C. Barnhart*

# *Network Fleet Assignment Objective Function*

- **For each fleet type/flight combination:**

  **Assignment Cost ≡ Operating cost + Spill cost**

- **Operating cost of assigning a fleet type *k* to a flight leg *j* is relatively straightforward to compute**
  - Can capture range restrictions, noise restrictions, water restrictions, etc. by assigning “infinite” costs
- **Spill cost for flight leg *j* and fleet assignment *k***
  - Average revenue per passenger on *j* * MAX(0, unconstrained demand for *j* – number of seats on *k)*
  - But revenue for each flight leg is affected by fare class mix (RM) as well as itinerary mix in a network

## *Constraints*

- **Cover Constraints**
  - Each flight must be assigned to exactly one fleet type
- **Balance Constraints**
  - Number of aircraft of a fleet type arriving at a station must equal the number of aircraft of that fleet type departing
- **Aircraft Count Constraints**
  - Number of aircraft of a fleet type used cannot exceed the number available

# *Network Fleet Assignment Formulation*

## Formulation: The Fleet Assignment Model (FAM)

**maximize**

***PROFIT***

**subject to**

***Flight Cover***

***Aircraft Balance***

***Aircraft Count***

***Integrality and Non-negativity***

*Ref.: Hane C., C. Barnhart, E. Johnson, R. Marsten, G. Nemhauser, G. Sigismondi. 1995. The Fleet Assignment Problem: Solving a Large-Scale Integer Program. Mathematical Programming 70 211-232.*

# *Solution*

- **Solve fleet assignment problems for large network carriers (10-14 fleets, 2000-3500 flights) within 10-20 minutes of computation time on workstation class computers**

- **Hane, et al. "The Fleet Assignment Problem, Solving a Large Integer Program," *Mathematical Programming*, Vol. 70, 2, pp. 211-232, 1995**

# *COLDSTART:*
# *Fleet Assignment at Delta Air Lines*

- **1994 Interfaces article describes implementation of large-scale network fleet assignment optimization**
  - 2500 domestic flight legs per day
  - 450 aircraft of 10 different fleet types
- **Mixed-integer linear program**
  - Minimize assignment costs over the Delta domestic network and schedule for one day
  - Assigns fleet types to each leg, not tail numbers (aircraft routing performed subsequently)
- **First OR application of this size implemented at Delta**
  - Use of this model estimated to increase operating profit by $100 million per year

# *Constraints and Issues in Coldstart*

- **Certain pairs of legs must be assigned same fleet**
  - Provide one-stop, same-plane service through the hub
  - Tag-end flights that must be operated with same aircraft (e.g., IST-GRU-EZE)
- **Model includes maintenance requirements**
  - Use “maintenance arcs” to represent flights that must be covered with an aircraft
  - For example, a B757 must be at a designated base each night
- **Crew considerations**
  - Common fleet families use same pilot aggregates
  - Penalize fleet assignments that require extended crew rest periods – for example, when only 1 flight into/out of a city is assigned an aircraft type

# *Constraints and Issues in Coldstart*

- **Aircraft performance can differ within each fleet type**
  - Different engines; take-off and landing weights
  - Not all aircraft equipped to serve over-water routes
- **Airport characteristics and restrictions**
  - Runway lengths and temperature limitations
  - Certain airports have noise restrictions and/or curfews for specific aircraft types
- **Assumed turn times determine aircraft availability**
  - Minimum turn around times vary by both aircraft type and airport
  - Larger aircraft require longer turn times
  - International flights require longer turn times than domestic flights with same fleet type

# *Coldstart in the Scheduling Process*

- **Changes to the functions of DL schedule planners**
  - Model performs assignments to schedule, while planner reviews and analyzes the impacts of assignment changes
  - Focus on cost and revenue inputs to the model instead of the actual optimization process

- **Model allows for what-if analysis by planners**
  - Compare results from two different optimization runs to compare impacts of various costs and constraints
  - Evaluate the changes required in order to change the aircraft assigned to a particular flight leg – up-line and down-line swaps required